Key

Math 4 Name ________________

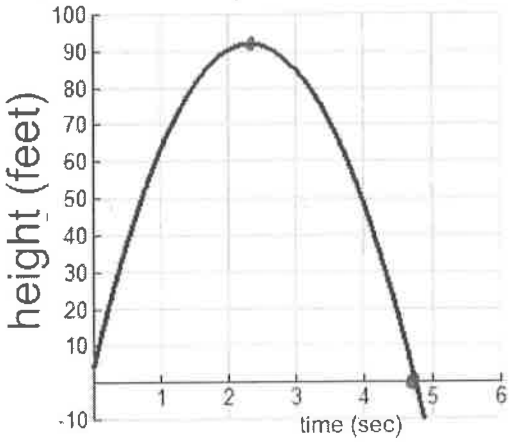
5-5 Acceleration and Deceleration Date________

In this activity you will be working towards the following learning goals:

I can use derivatives to find the velocity and acceleration of a moving object and solve problems involving particle motion.

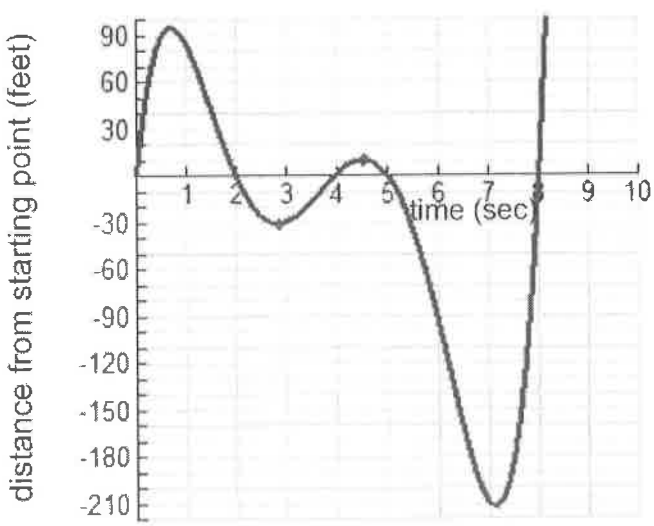
Introduction

Below is the height of a projectile (*in feet*) over time (*in seconds*) launched into the air



When is the projectile *speeding* up? $2.2 < x < 4.7$ sec

When is the projectile *slowing* down? $0 < x < 2.2$ sec



When is the particle *speeding up?*

$0.8 < x < 2$ sec

$2.9 < x < 3.7$

$4.5 < x < 6$ $x > 7.1$

When is the particle *slowing down?*

$0 < x < 0.8$ $6 < x < 7.1$

$0 < x < 2.9$

$3.7 < x < 4.5$

If an object is traveling at a velocity of -200 ft/sec , what does the negative sign imply?

The object is traveling down / backward / back to its starting point.

If the object's velocity gets "more negative", what does that mean about the object?

It is traveling faster in the "negative" direction.

If the object's velocity gets "less negative", what does that mean about the object?

Still traveling in the negative direction, but as a slower speed.

Applications of the Derivative Rules

Example #1: Projectile Motion

The position of a projectile (in feet) after t seconds is given by the equation:

$$h(t) = -16t^2 + 800t - 3$$

a. Find a formula for the instantaneous velocity of the projectile.

$h'(t) = v(t) = -32t + 800$

b. Find the instantaneous velocity of the projectile when $t = 0$, 5, 10, 15 and 20 seconds.

$v(0) = -32(0) + 800 = 800$ ft/s

$v(5) = 640$ ft/s

$v(10) = 480$ ft/s

$v(15) = 320$ ft/s

$v(20) = 160$ ft/s

c. Find the following average rates of change in ***velocity***: $t = 0$ to $t = 5$; $t = 5$ to $t = 10$; $t = 15$ to $t = 20$

Include units!!

t	h'(t)
0	800 ft/sec
5	640 ft/sec
10	480 ft/sec
15	320 ft/sec
20	160 ft/sec

Rate of change in velocity from $t = 0$ to $t = 5$: $\frac{640-800}{5-0} = \frac{-160}{5} = -32 \frac{\text{ft/sec}}{\text{sec}}$

Rate of change in velocity from $t = 5$ to $t = 10$: $\frac{480-640}{10-5} = -32$ ft/sec/sec

Rate of change in velocity from $t = 10$ to $t = 15$: $\frac{320-480}{15-10} = -32$ ft/sec^2

Rate of change in velocity from $t = 15$ to $t = 20$: $\frac{160-320}{20-15} = -32$ ft/s^2

In part *c* of the previous example, you calculated the **acceleration** of the object. Complete the following statements about acceleration.

1. Acceleration is the rate of change of a rate of change.
2. Acceleration describes how fast the velocity is changing.
3. When a car accelerates, its velocity increases.
4. When acceleration decreases, it is called deceleration.
5. When a car decelerates, its velocity decreases.
6. The **instantaneous acceleration *a(t)*,** of a projectile at time *t* is defined as the instantaneous rate of change of its velocity with respect to time at time *t*, and can be calculated by taking the derivative of the velocity function.

In general, **position *h(t)*, velocity *v(t)*,** and **acceleration *a(t)*,** are related as follows:

Velocity is the derivative of position.

Acceleration is the derivative of velocity.

∴ acceleration is the **second derivative** of position.

In symbols, this looks like:

position = $h(t)$

$v(t) = h'(t)$

$a(t) = h''(t)$

Example #2: Velocity & Acceleration of an Object

An object moves so that its position (in meters) at time t seconds is given by the function:

$$p(t)=\frac{2}{3}t^3-8t^2+5t+7$$

a. Find the function that represents the velocity of the object.

$p'(t)=v(t)=2t^2-16t+5$

b. Find the velocity of the object when $t=7$ second; at $t=9$ seconds.
Interpret the meaning of your calculations.

Velocity at t = 7 is -9 ft/s. This means... the object is moving backward at a speed of 9 ft/s after 7 seconds.

Velocity at t = 9 is 23 ft/s. This means... after 9 seconds, the object is moving forward at a speed of 23 ft/s.

c. Find the function that represents the acceleration of the object.

$p''(t)=a(t)=4t-16$

d. Find the acceleration when $t=7$ second; at t = 9 seconds seconds.
Interpret the meaning of your calculation.

Acceleration at t = 7 is 12 ft/sec². This means... the velocity of the object is increasing by 12 ft/s every second after 7 seconds.

Acceleration at t = 9 is 20 ft/sec². This means... at 9 seconds, the velocity of the object is increasing by 20 ft/s each second.

e. At 9 seconds, is the object speeding up or slowing down? At 7 seconds? How do you know?

The object is speeding up at 9 seconds. Both the velocity and acceleration are positive. The object is slowing down at 7 seconds. The velocity is negative but increasing (acceleration is positive).

f. In general, how can you tell if an object is speeding up or slowing down based on its velocity and acceleration?

If the velocity + acceleration have opposite signs, the object is speeding up or slowing down.

Example #3: Particle Motion

A particle is moving along the horizontal axis in such a way that its position at time t is given by the following function:

$$s(t) = t^3 - 6t^2 + 9t, \; (0 \le t \le 5) \quad \text{Domain}$$

a. Determine a formula for the velocity of the particle.

$S'(t) = V(t) = 3t^2 - 12t + 9$

b. Determine a formula for the acceleration of the particle.

$S''(t) = a(t) = 6t - 12$

c. For what values of t is the particle at rest? *Hint: What is its velocity when it's at rest?*

$0 = 3t^2 - 12t + 9$

$0 = 3(t^2 - 4t + 3)$

$0 = 3(t-3)(t-1)$

$\boxed{t = 1 \text{ second}, \quad t = 3 \text{ seconds}}$

d. When is the particle moving to the right? To the left? *Hint: NUMBER LINE ANALYSIS!*

$V(t) = 3(t-3)(t-1)$

	$t<1$	$1<t<3$	$t>3$
$t-3$	$-$	$-$	$+$
$t-1$	$-$	$+$	$+$
$V(t)$	$+$	$-$	$+$
	Right	Left	Right

Right: $0 \le t < 1$ & $3 < t \le 5$

Left: $1 < t < 3$

e. What is the velocity of the particle when the acceleration is zero?

$0 = 6t - 12$
$12 = 6t$
$t = 2$

$V(2) = 3(2)^2 - 12(2) + 9 = \boxed{-3}$

f. What is the position of the particle at $t = 4$ seconds?

$S(4) = 4^3 - 6(4)^2 + 9(4) = \boxed{4}$

g. When $t = 3$, what is the total distance traveled by the particle? *Refer back to your NLA analysis for help!*

$S(0) = 0 \rightarrow$ Beginning

$S(1) = 4$ m after moving to the right for 1 second (from 0-1 seconds)

$S(3) = 0$ after moving to the left for 2 seconds (from 1-3 seconds)

4 m $\longrightarrow$

$\longleftarrow$ 4 m

Overall position is back at starting point (0).

Distance covered: $\boxed{8\text{m}}$